International Journal of Electronics Engineering, 1(2), 2009, pp. 235-237

System Identification Using Output Error Method in the Presence of Uncertainities

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Abstract: *In the robust performance analysis of a given controller, the choice for an appropriate model uncertainty structure is important. From an identification experiment the measurement data is typically mapped into an uncertainty set that is represented in a particular structure e.g., norm-bounded additive perturbation.Amongst such a variety of possible uncertainty structures an ultimate question to be answered would be what is, for a given purpose (robust stability/performance analysis), the best model uncertainty structure in which to identify the model set (nominal model and uncertainty bound). And consequently, what would be the best experiment allowing for minimization of the uncertainty. In this paper the analysis of the robust adaptive systems identification algorithms has been carried out in the presence of additive noise and magnitude bounded perturbations.*

Keywords: *Adaptive ARMA System Identification, Uncertainty Structures, Gaussian Noise, Norm-bounded Perturbation, Median Smoothing, Adaptive Output Error.*

INTRODUCTION

Fundamental connections between adaptive filtering, identification, and control have been widely acknowledged since their inception in the late 1960s and consequently the adaptive systems identification has been an active area of research [1]. In this paper, we only consider the linear adaptive ARMA system identification [2, 3]. Fundamentally there have been two approaches to adaptive filtering that correspond to different formulations of the prediction error; these are known as equation error and output error methods. It is well known fact that when data is contaminated by Gaussian noise conventional linear systems may perform poorly. The linear filter or linear smoothing has no robustness to wild-points assisted in the additive noise, for the purpose of robustness of adaptive system identification; the nonlinear *M*-estimate should be used in data processing [4].

In this paper, adaptive ARMA system identification is realized by the adaptive output error algorithm [5] that is given in Section 2. The choice for an appropriate model uncertainty structure is important in the robustness analysis. An amplitude-bounded (circular) uncertainty set can equivalently be described in terms of an additive, Youla parameter and γ-gap uncertainty [6]. A brief introduction to various uncertainty structures is given in Section 3. In Section 4, results obtained by adaptive output error algorithm by using median smoothing for identification of a nominal model and for a nominal model with amplitude bounded additive uncertainty are presented followed by conclusions in Section 5.

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ROBUST ADAPTIVE OUTPUT ERROR SYSTEM IDENTIFICATION ALGORITHM

In general, the adaptive ARMA system identification includes equation error and output error algorithm. In this section, we will discuss the robust adaptive output error ARMA system identification algorithm [2].

A single input and single output ARMA system can be written as

$$A(q)d'(t) = B(q)u(t)$$

$$A(q) = 1 - \sum_{i=1}^{N} a_i q^{-i}, \ \ B(q) = \sum_{j=0}^{M-1} b_j q^{-j} \tag{1}$$

$$d(t) = d'(t) + \mathrm{n}(t)$$

Where $d(t)$, $u(t)$ are the output and input respectively, $n(t)$ denotes the additive noise.

Considering the ARMA system (1) its predictor is of the form,

$$\hat{y}(t) = \sum_{i=1}^{N} \hat{a}_i y(t-i) - \sum_{j=0}^{M-1} b_j u(t-j) \tag{2}$$

Considering the output error:

$$e(t) = d(t) - y(t) = H_1(q)[D'^T(t)\omega(t)] + n(t-1)$$

Where $H_1(q) = \dfrac{1}{1 - \sum_{i=1}^{n} \hat{a}_i q^{-i}}$ and

$$D'(t) = [d'(t), \ldots, d'(t-N), u(t), \ldots, u(t-M+1)]^T$$

By using the output error $e(t)$, the robust adaptive output error ARMA system identification algorithm is derived as:

$$\hat{\omega}(t) = \hat{\omega}(t-1) + R(t)e(t)H_2(q)[D_r(t)] \tag{3a}$$

$$e(t) = d_r(t) - y(t) \tag{3b}$$

$$d_r(t) = R(t)d(t) \tag{3c}$$

$$\hat{A}(q)y(t) = \hat{B}(q)u(t) \tag{3d}$$

$$H_2(q) = H_1^{-1}(q) = \hat{A}(q) \tag{3e}$$

$$D_r(t) = [d_r(t-1),....,d_r(t-N),u(t),.....,u(t-M+1)]^T$$

Where $R(t)$ is the median smoothing influence function which cancel the influence of wild-points in $n(t)$.

$$R(t) = \begin{cases} 1, & if\ |d(t) - d_m(t)| \le T; \\ \dfrac{K}{|d(t)|}, & otherwise. \end{cases} \tag{3f}$$

$$d_m(t) = median[d(t-1),.....,d(t-L)]$$

T is the threshold, selected as:

$$T = (3 \sim 5)median[|d(t) - d_m(t)|,......,|d(t-L) - d_m(t)|]$$

From the adaptive algorithm (3a)–3(f), The estimate 3(d) of ARMA system is a recursive form, thus, the system may be unstable unless the roots of $\hat{A}(q) = 0$ lie inside unit circle during the adaption. The wild-points existed in $d(t)$ may influence the parameter estimate $\hat{\omega}(t)$, greatly, and make the adaptive algorithm divergence, if $d(t)$ is not prefiltered by the median smoothing. Hence, it is significant to cancel the influence of wild-points using median smoothing to keep the adaptive algorithm stability. If $R(t)$ takes the form of (3f), it is obvious that the ARMA system output $d(t)$ contaminated by the wild-points is prefiltered [7, 8] or smoothed through the nonlinear median smoothing, the influence by the wild-points is cancelled.

Framework for Uncertainties

Considering single-input–single-output linear time invariant finite-dimensional systems *G(s)* and controllers *C(s)*. Co-prime factorizations of plants and controllers are defined as $G(s)=N(s)D^{-1}(s)$ and $C(s) = N_c(SD_c^{-1}(s))$ where $N(s)$, $D(s)$, $N_c, D_c \in \mathbb{R}\ H\infty$ satisfy the usual conditions [6, 9]. The factorizations are normalized, denoted by $\overline{(.)}$, if they additionally satisfy $\bar{N}(s)*\bar{N}(s)+\bar{D}(s)=1$, where (.)* denotes complex conjugate transpose. This paper considers three model sets based on a specific uncertainty structure.

Additive Uncertainty:

$$g_a(G_x, W_a) := \{G_\Delta(s) \mid G_\Delta(s) = G_x(s) + \Delta_a(s), |\Delta_a(i\omega)| \ \forall\omega \in \mathbb{R}\}, \tag{5}$$

With $G_x(s)$ is a nominal model and $W_a(s)$ a weighting function.

Youla-uncertainty:

$$g_Y(G_x, C, Q, Q_c, W_Y) := \left\{ G_\Delta(s) \middle| G_\Delta(s) = \frac{\bar{N}_x(s) + \bar{D}_c(s)\Delta_G(s)}{\bar{D}_x(s) + \bar{N}_c(s)\Delta_G(s)}, \right. \left. |Q_c^{-1}(i\omega)\Delta_G(i\omega)Q(i\omega)| \le |W_Y(i\omega)| \ \forall\omega \in \mathbb{R} \right\} \tag{6}$$

With $G_x(s) = \bar{N}_x(s)\bar{D}_x^{-1}(s)$ a nominal model, $C(s) = \bar{N}_x(s)\bar{D}_x^{-1}(s)$ a present controller and Q_s, $Q_c(s)$ stable and stably invertible weighting functions reflecting the freedom in choosing the Co-prime factorization of $G_x(s)$ and $C(s)$ [6]. An additional weighting can be provided by $W_Y(s)$. The Youla parameter $\Delta_G(s)$ is uniquely determined by:

$$\Delta_c(s) = \bar{D}_c^{-1}(s)(1 + G_\Delta(s)C(s))^{-1}(G_\Delta(s) - G_x(s))\bar{D}_x(s).$$

g-gap uncertainty:

$$g_Y(G_x, W_Y) := \{G_\Delta(s) | k(G_\Delta(i\omega), G_x(i\omega)) \le |W_Y(i\omega) \forall\omega \in \mathbb{R}\} \tag{7}$$

With $k(G_\Delta, G_x)$ the chordal distance between a plant $G_\Delta(s) = \bar{N}_\Delta(s)\bar{D}_\Delta^{-1}(s)$ and the nominal model $G_x(s) = \bar{N}_x(s)\bar{D}_x^{-1}(s)$, defined by

$$K(G_\Delta(i\omega), G_x(i\omega)) := |\bar{N}_x(i\omega)\bar{D}_\Delta(i\omega) - \bar{D}_x(i\omega)\bar{N}_\Delta(i\omega)| = \frac{|G_x(i\omega) - G_\Delta(i\omega)|}{\sqrt{(1 + |G_\Delta(i\omega)|^2)(1 + |G_x(i\omega)|^2)}}.$$

At this point pole/zero conditions are not yet imposed on $G_\Delta(s)$, $G_x(s)$, $\Delta(s)$, $P(s)$ or $W(s)$ as required when studying robust stability conditions [9, 10]. The focus lies here with the properties of the frequency responses of the sets.

Case Study

A) The robust adaptive output error ARMA system identification (3a)-(3f) is used in the following simulation [11]. Its parameter estimate will be compared with that the conventional adaptive output error algorithm without median smoothing, The ARMA system used in the simulation is of the form,

$$A(q) = 1 + .3q^{-1} - .4q^{-2}$$
$$B(q) = 1 - 1.4q^{-1}$$

The additive noise $n(t)$ takes the following form:

$$n(t) = \begin{cases} n_1(t), & if\ a \ge 0.9; \\ n_2(t) & otherwise. \end{cases}$$

Where $n_1(t)$ the Gaussian is white noise with variance 0.01, and $n_2(t)$ is also the white noise with variance 1. a is a random, its distribution is uniform within [0,1].

Compared with the conventional adaptive algorithm, the robust adaptive algorithm is much robust to wild points; its robust performance is improved, greatly.

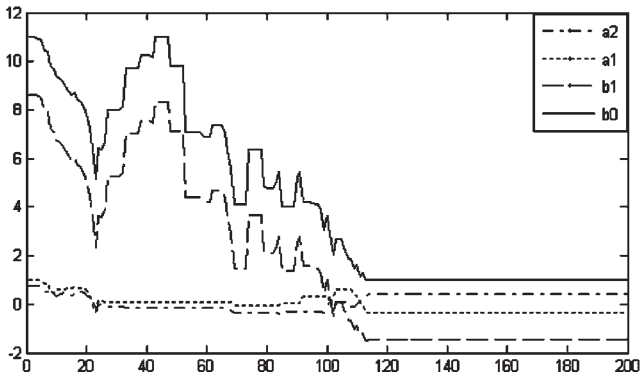

Figure 1: The Parameter Estimate in the presence of Noise without Upper Bound.

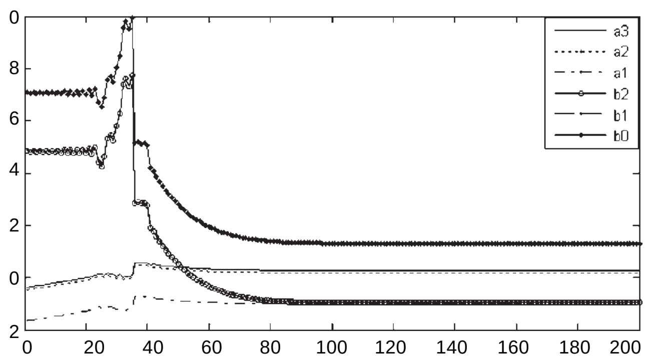

Figure 2: The Parameter Estimate in the presence of Noise with Additive Uncertainty

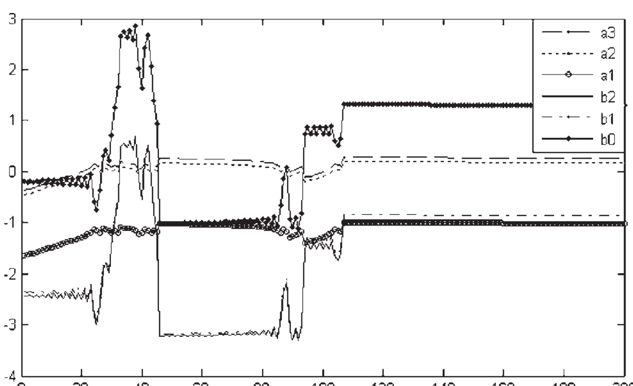

Figure 3: The Parameter Estimate in the presence of Noise with Youla Uncertainty

B) Now the simulation results are obtained for amplitude bounded additive uncertainty model structure and the model now taken is of the form similar to that used in [12]

$$G(z) = \frac{B(z) + \Delta(z)D(z)}{A(z)}$$

Where $D(z) = \dfrac{.32(1-.5z^{-1})}{1+.7z^{-1}}$

With $\|\Delta(z)_{Hx}\| \le 1$

It is seen that convergence is obtained with additive uncertainty in the plant.

C) Again the simulation results are obtained for amplitude bounded youla uncertainty model structure and the model taken is of the form similar to that used in [12]

With $\|\Delta(z)_{Hx}\| = .968$

It is seen that convergence is again obtained with youla uncertainty in the plant.

CONCLUSION

In this paper, a robust adaptive system identification algorithm which has great robustness to wild-points have been analyzed. The amplitude bounded non-parametric uncertainties are accounted for by including a model of the uncertainty in the identifier. It is seen that when uncertainty is incorporated in the plant model, the convergence rate of parameters is observed. Further work can be carried out by describing uncertainty set in terms of γ-gap.

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